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SOME LOCAL MODELS FOR CORRELATION EXPERIMENTS*

ABSTRACT. This paper constructs two classes of models for the quantum correlation experiments used to test the Bell-type inequalities, synchronization models and prism models. Both classes employ deterministic hidden variables, satisfy the causal requirements of physical locality, and yield precisely the quantum mechanical statistics. In the synchronization models, the joint probabilities, for each emission, do not factor in the manner of stochastic independence, showing that such factorizability is not required for locality. In the prism models the observables are not random variables over a common space; hence these models throw into question the entire random variables idiom of the literature. Both classes of models appear to be testable.

1. INTRODUCTION

I want to call attention to the possibility that the quantum mechanical statistics, found in the various correlation experiments devised to test the Bell-type inequalities, may be accounted for by means of statistical models of the experiments that are local and realistic. (This is the terminology of Clauser and Shimony (1978). The phrase "local hidden variables" is more commonly used.) Despite the various and sometimes elegant derivations of the Bell inequalities, which conflict with the quantum mechanical statistics, the possibility for such local and realistic accounts arises in two ways. First, as I pointed out several years ago (1974), the various derivations of these inequalities invariably rely on background assumptions beyond those of realism and locality. Hence it may be possible to build local and realistic models based on different background assumptions. Second, the application of the Bell results to real experiments always involves special assumptions about the experimental processes. Hence different assumptions may undercut the inequalities while remaining within the framework of locality and realism. The "conspiratorial selection" model proposed for the photon correlation experiments by Clauser and Horne (1974) (and quickly rejected by them as "unnatural") exploits this second way.

My purpose here is to outline two classes of models, *prism models*

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and *synchronization models*, which are local and realistic and which, nevertheless, by taking advantage of the two indicated ways (respectively), yield the quantum mechanical statistics. I offer these models in a Popperian spirit, that is, as conjectures to be further elaborated, criticised, and tested. I hope that some of the workers in this area who find the Bell results compelling, if not conclusive, will find these models an interesting enough challenge to merit response.

2. BACKGROUND AND TERMINOLOGY

The ideal sort of experiment outlined in Clauser and Horne (1974) seems broad enough to encompass the various real experiments performed or contemplated. It consists of a source that emits two-particle systems, where each composite system is in one and the same "singlet" state ψ. The "particles" (I shall refer to them as A and B) are emitted in opposite directions (these define the A-wing and the B-wing of the experiment). In each wing there is an analyser that may be set in various positions. It is convenient to take these positions as co-planar angles, relative to some fixed direction, in the plane transverse to the "path" of the particles. The analyser is followed by a detector which, if triggered, will count the presence of a particle of the sort emitted by the source. (I speak here of "particles" and their "paths" and below of "particles passing an analyser and being detected". In the case of photons – or bosons more generally – this language and imagery is out of place. It is a convenient metaphor, however, and I use it for that reason and in the belief that it does not mask any objectionable features of the models.)

The models I shall propose are not designed as general accounts of spin or polarization and their measurements, or the like. They are designed to account for the statistics of experiments, like that outlined above, in the following more restrictive sense. Given the detailed plan for an experiment (already run or contemplated) – that is, the specific geometry of the experimental arrangement, the sequence of analyzer positions and the nature of the source – the models postulate a particular statistical distribution of the particle-pairs that gives rise to the quantum mechanical probabilities for the experimental outcomes. Thus what I call "models" are perhaps better thought of as model *schemas* that produce specific statistical models for particular experimental designs: feed the design into the schema and out comes a model of the experiment.

The experiments designed to test the Bell inequalities involve frequency measurements for only two distinct relative orientations of the analyzers. They employ orientations of the A-analyzer in two (generic) positions A_1 and A_2, and orientations of the B-analyzer in two (generic) positions B_1 and B_2, such that if θ_{ij} is the magnitude of the angle between the i^{th} position of the A-analyzer and the j^{th} position of the B-analyzer then

$$\theta_{11} = \theta_{21} = \theta_{22} = \theta \qquad \text{and} \qquad \theta_{12} = \theta' \neq \theta.$$

The design and implementation of such an experiment then involves fixing the relative orientations θ and θ' (e.g., for the photon correlation experiments one wants $\theta' = 3\theta = 3\pi/8$ for the most severe test), and then producing a number of orientations for the A-analyzer to correspond to its first position, a number of orientations for its second position – and similarly for the two generic B positions – which give the relative orientations θ, θ' as prescribed. Thus I shall speak of "one of the first positions of A" and "one of the second positions of B" (etc.), where I refer to a relative orientation of magnitude $\theta_{12} = \theta'$ (etc.).

I shall assume that the observed statistics in such experiments approximate the probabilities of quantum mechanics derived from the composite state function ψ, and which I designate as follows, for $i = 1, 2$; $j = 1, 2$.

P_{A_i} = the probability for a count in the A-wing when the A-analyzer is set in one of its i^{th} positions.

P_{B_j} = the probability for a count in the B-wing when the B-analyzer is set in one of its j^{th} positions.

$P_{A_iB_j}$ = the probability for a coincidence count when the A-analyzer is set in one of its i^{th} positions and the B-analyzer is set in one of its j^{th} positions.

In the usual cases of interest one has, from the symmetry of the composite state ψ, that

$P_{A_i} = 1/2 = P_{B_j}$ for all i and j, and that $P_{A_1B_1} = P_{A_2B_1} =$

$P_{A_2B_2} = P_\theta$ and $P_{A_1B_2} = P_{\theta'}$ where $0 \leq P_\theta \leq 1/2$, $0 \leq P_{\theta'} \leq 1/2$ and $P_\theta \neq P_{\theta'}$.

I shall assume these relations here, although the reader will be able to see how to modify the proposed models in cases where such symmetry considerations do not apply.

To satisfy the requirement of realism (i.e., of hidden variables), suppose that the particle pairs emitted by the source come in various *types* λ, where λ ranges over some set L of real numbers and is distributed according to some (normalized) continuous density function ρ. The particles in a pair of type λ – which I shall refer to as being themselves of type λ – will be supposed already to possess certain properties that dispose them either to pass an analyzer set in certain positions and then to be detected, or not. Moreover, to satisfy locality, suppose that this disposition applies to each particle individually, without regard to the circumstances of the other particles whether of the same or of different types.

So far it may appear that I have merely redescribed the standard framework of (deterministic) local hidden variables, as discussed in the Bell literature. I have, however, been careful not to include an assumption implicit in that literature, namely, that each particle can respond to the analyzer detector assembly in its wing when the analyzer is set in *any* position whatsoever. The form this assumption generally takes is to suppose that the probability to be detected when the analyzer is set in a given position is well-defined, for each particle and for each position. If one thinks of probabilities realistically, however, then they should be grounded in physical properties. Where the requisite physical properties are not present, no question of probability arises – not even probability zero. (Thus one would not talk about the probability for a liquid to be harder than a gas, nor would one say that such a circumstance has probability zero.) Since the types λ code for certain physical properties, one can make them code as well for those properties that make a particle suitable for certain measurements (and not others).

Thus for each of the generic analyzer positions A_i, B_j associate the set $\sigma(A_i)$, $\sigma(B_j)$ consisting of those types λ which are suitable for the designated measurement in this position. That is,

$\lambda \epsilon \sigma(A_i)$ iff there is some probability (perhaps zero) that an

> A-particle of type λ will pass an A-analyzer set in one of the i^{th} positions and then be counted.

(Similarly for $\sigma(B_j)$.) So if $\lambda \notin \sigma(A_i)$, then ascribing a probability for an A-particle of type λ to pass an A-analyzer set in one of its i^{th} positions and then to be detected is, as in the illustration about liquids being harder than gases, to commit something like a category mistake. Hence I shall call $\sigma(A_i)$, $\sigma(B_j)$ the *category* of A_i, B_j.

3. PRISM MODELS

The Bell literature has supposed that the category for each and every measurement is one and the same set, the whole λ-space L. Arguing from this assumption of one common category, these authors find that the statistics for certain correlated systems cannot be generated by a local and realistic model. Just as Occam's Razor cautions that it is vain to do with more what can be done with less, so Karl Menger (1960) dubs a "prism principle" the opposite maxim, that it is vain to try to do with fewer what requires more. It would appear that local and realistic ways of generating the statistics for correlated systems require more than one common category. Thus I call such ways *prism models*.

The category of a measurement specifies for which types of particles the measurement is suited. The particular measurement outcome, for a suitable particle, is then fixed by the properties encoded in the type of the particle. One can exhibit this encoding by defining, for suitable particles, a function that tells how the particles are going to respond to measurement. Thus introduce *response functions* $A_i(\lambda)$, $B_j(\lambda)$ whose significance is as follows. *For* $\lambda \epsilon \sigma(A_i)$,

> $A_i(\lambda) = 1$ iff an A-particle of type λ will produce a count in the A-wing when the A-analyzer is in one of its i^{th} positions.
>
> $A_i(\lambda) = 0$ iff $A_i(\lambda) \neq 1$ (and $\lambda \epsilon \sigma(A_i)$).

A similar significance attaches to the $B_j(\lambda)$ response functions.

In the standard Bell literature these response functions are defined for all λ. They then become random variables with respect to the density ρ. Bell and others suppose that the correlation statistics should be derived from the joint distributions of these random variables, and the various inequalities show that this project cannot be carried out satisfactorily. It is this framework of random variables and their joint distributions that I have criticized (Fine, 1974)[1]. This apparatus is avoided in the prism models because the response functions are defined only on the corresponding categories and hence not for all λ. Thus the response functions are not random variables and there is no question of getting out the correlation statistics from their joint distributions, since these latter need not be well-defined. How, then, are we to compute the probabilities for single and coincident counts?

The answer seems clear enough. The probability for a certain measurement result must be the probability that a system will produce that result, given that the system is of a category suited to the measurement. Thus prism models should generate the quantum mechanical probabilities as conditional probabilities, conditional on the various categories of measurements. Specifically we require that

$$P_{A_i} = \mathrm{Prob}[A_i(\lambda) = 1 \big| \lambda \epsilon \sigma(A_i)],$$

$$P_{B_j} = \mathrm{Prob}[B_j(\lambda) = 1 \big| \lambda \epsilon \sigma(B_j)], \text{ and}$$

$$P_{A_iB_j} = \mathrm{Prob}[A_i(\lambda) = B_j(\lambda) = 1 \big| \lambda \epsilon \sigma(A_i) \cap \sigma(B_j)].$$

Using the λ-density ρ we can rewrite these in standard fashion as ratios of integrals.

It remains to show that there exist prism models reproducing the quantum probabilities as above. In fact, there are many such models. Here, I want to outline two. The first is, perhaps, the most simple and least interesting. I call it *the minimal model.*

To construct the minimal model, let the range L of λ be the closed interval $[0, 4]$ and suppose that λ is distributed uniformly there; i.e., that $\rho(\lambda) = 1/4$. Let the categories be assigned as follows: $\sigma(A_1) = [0, 2]$, $\sigma(A_2) = (2, 4]$, $\sigma(B_1) = [0, 1) \cup (2, 3]$ and $\sigma(B_2) = [1, 2] \cup (3, 4]$. (Note that the categories corresponding to incompatible measurements – A_1, A_2 and B_1, B_2 – are disjoint. That is the root idea of the mimimal model.) Next, consider step functions f, g_ϕ defined on $[0, 1]$

as follows.

$$f(x) = \begin{cases} 1, 0 \leq x \leq 1/2 \\ 0, 1/2 < x \leq 1 \end{cases}$$

and

$$g_\phi(x) = \begin{cases} 1, 0 \leq x \leq P_\phi \\ 0, P_\phi < x \leq 1/2 \\ 1, 1/2 < x \leq 1 - P_\phi \\ 0, 1 - P_\phi < x \leq 1 \end{cases}$$

for $\phi = \theta$ or $\phi = \theta'$.

Notice that

$$\int_0^1 f(x)\, \mathrm{dx} = \int_0^1 g(x)\, \mathrm{dx} = 1/2;$$

and

$$\int_0^1 f(x) g_\phi(x)\, \mathrm{dx} = P_\phi.$$

These functions will now be used to define the response functions $A_i(\lambda)$ and $B_j(\lambda)$ in the following manner.

$$A_1(\lambda) = \begin{cases} f(\lambda), 0 \leq \lambda < 1; \\ f(\lambda - 1), 1 \leq \lambda \leq 2. \end{cases}$$

This defines $A_1(\lambda)$ on $\sigma(A_1)$.

$$A_2(\lambda) = \begin{cases} f(\lambda - 2), 2 < \lambda \leq 3; \\ f(\lambda - 3), 3 < \lambda \leq 4. \end{cases}$$

This defines $A_2(\lambda)$ on $\sigma(A_2)$.

$$B_1(\lambda) = \begin{cases} g_\theta(\lambda), 0 \leq \lambda < 1; \\ g_\theta(\lambda - 2), 2 < \lambda \leq 3. \end{cases}$$

This defines $B_1(\lambda)$ on $\sigma(B_1)$.

$$B_2(\lambda) = \begin{cases} g_{\theta'}(\lambda - 1), 1 \leq \lambda \leq 2 \\ g_{\theta}(\lambda - 3), 3 < \lambda \leq 4 \end{cases}$$

This defines $B_2(\lambda)$ on $\sigma(B_2)$.

One can now calculate and see that the quantum probabilities are the required conditional probabilities. For example,

$$\text{Prob}[A_1(\lambda) = 1 | \lambda \epsilon \sigma(A_1)] = \frac{1/4 \left[\int_0^1 f(\lambda)\,\mathrm{d}\lambda + \int_1^2 f(\lambda - 1)\,\mathrm{d}\lambda \right]}{1/4 \int_0^2 \mathrm{d}\lambda}$$

$$= \frac{1/2 + 1/2}{2} = 1/2 = P_{A_1}.$$

A typical calculation for the joint distributions goes like this. Notice that $\sigma(A_1) \cap \sigma(B_2) = [1, 2]$, so

$$\text{Prob}[A_1(\lambda) = B_2(\lambda) = 1 | \lambda \epsilon \sigma(A_1) \cap \sigma(B_2)] =$$

$$\frac{1/4 \int_1^2 f(\lambda - 1) g_{\theta'}(\lambda - 1)\,\mathrm{d}\lambda}{1/4 \int_1^2 \mathrm{d}\lambda} = P_{\theta'} = P_{A_1 B_2}.$$

In the minimal model each particle is targeted to be responsive to exactly one analyzer position. One can do better than that. I believe the best one can do, in this setting, without letting in enough apparatus to derive the Bell inequalities, is captured by the *maximal model* which I shall proceed to outline. It is motivated by the following reflections. The Wigner version of the Bell-inequalities makes plain that the deviation from quantum mechanics comes from the assumption that all the response functions (in my terminology) are defined for all λ. For this leads to there being well-defined multiple distributions, like $P_{A_1 A_2 B_1 B_2}$ which are never well-defined in quantum theory itself and from which the Bell inequality follows immediately. (For this way of viewing Wigner's proof see my (1974).) It can happen, however, that distributions like $P_{A_1A_2B_1}$ *are* well-defined quantum mechanically, even though A_1 and A_2 are incompatible. For instance, if A_1 were strictly correlated with B_1, then one would have that $P_{A_1A_2B_1} = P_{A_2B_1}$. Hence the best one can have, without presupposing something in direct conflict with quantum mechanics, is that three of the four response functions may be defined for certain measurable sets of λ (but not all four at once). This is the guiding idea for the *maximal model.*

To construct it suppose that, as before, L is $[0, 4]$ and ρ is the uniform distribution. Specify the categories as follows:

$$\begin{aligned}
\sigma(A_1) &= [0, 1) \cup (2, 4] \\
\sigma(A_2) &= [1, 2] \cup (2, 4] \\
\sigma(B_1) &= (2, 3] \cup [0, 2) \\
\sigma(B_2) &= (3, 4] \cup [0, 2].
\end{aligned}$$

Using the step functions f, g_ϕ introduced for the minimal model, define the response functions as below:

$$A_1(\lambda) = \begin{cases} f(\lambda), 0 \le \lambda < 1 \\ g_\theta(\lambda - 2), 2 < \lambda \le 3 \\ g_{\theta'}(\lambda - 3), 3 < \lambda \le 4 \end{cases}$$

$$A_2(\lambda) = \begin{cases} f(\lambda - 1), 1 \le \lambda \le 2 \\ g_\theta(\lambda - 2), 2 < \lambda \le 3 \\ g_\theta(\lambda - 3), 3 < \lambda \le 4 \end{cases}$$

$$B_1(\lambda) = \begin{cases} f(\lambda - 2), 2 < \lambda \le 3 \\ g_\theta(\lambda), 0 \le \lambda < 1 \\ g_\theta(\lambda - 1), 1 \le \lambda \le 2 \end{cases}$$

$$B_2(\lambda) = \begin{cases} f(\lambda - 3), 3 < \lambda \le 4 \\ g_{\theta'}(\lambda), 0 \le \lambda < 1 \\ g_\theta(\lambda - 1), 1 \le \lambda \le 2. \end{cases}$$

Notice that each response function is defined on exactly three out of the four unit subintervals of $[0, 4]$ and, symmetrically, each λ has exactly three out of the four response functions defined on it. One can now readily calculate the appropriate conditional probabilities and see that, as required, they yield the probabilities prescribed by quantum mechanics. For instance,

$$\text{Prob}[B_1(\lambda) = 1 | \lambda \epsilon \sigma(B_1)] =$$

$$\frac{1/4\left[\int_2^3 f(\lambda - 2)\, d\lambda + \int_0^1 g_\theta(\lambda)\, d\lambda + \int_1^2 g_\theta(\lambda - 1)\, d\lambda\right]}{1/4\left[\int_0^3 d\lambda\right]}$$

$$= \frac{1/2 + 1/2 + 1/2}{3} = 1/2 = P_{B_1}.$$

And, since $\sigma(A_1) \cap \sigma(B_2) = [0, 1) \cup (3, 4]$,

$$\text{Prob}[A_1(\lambda) = B_2(\lambda) = 1 | \lambda \epsilon \sigma(A_1) \cap \sigma(B_2)] =$$

$$\frac{1/4\left[\int_0^1 f(\lambda) g_{\theta'}(\lambda)\, d\lambda + \int_3^4 f(\lambda - 3) g_{\theta'}(\lambda - 3)\, d\lambda\right]}{1/4\left[\int_0^1 d\lambda + \int_3^4 d\lambda\right]}$$

$$= \frac{P_{\theta'} + P_{\theta'}}{1 + 1} = P_{\theta'} = P_{A_1 B_2}.$$

There are prism models other than the maximal and minimal ones displayed above. It would be useful to have a systematic classification of them, which I do not have. Rather than produce any more, let me close the discussion of them with the reminder that they all come from the fundamental assumption that no system is capable of entering into a successful measurement interaction for every observable. Thus when we come to calculate probabilities as relative frequencies we should not employ the formula

$$\text{Prob}[A = x] = \frac{\text{number of systems measured where } A = x \text{ is found}}{\text{total number of systems measured}}.$$

Rather we want to calculate according to this formula,

$$\text{Prob}[A = x] =$$

$$\frac{\text{number of systems measured where } A = x \text{ is found}}{\text{total number of systems where a successful } A\text{-measurement is possible}}.$$

This latter formula results in the conditional probabilities of the prism models, and these, as we have seen, give the quantum mechanical correlations in a manner both realistic and local.

4. SYNCHRONIZATION MODELS

I have emphasized that at the root of the Bell results is the assumption that the observables (corresponding to the analyzer positions) be treated as random variables with respect to the λ-density, ρ. (In the finite frequency situation considered by Stapp (1971), ρ is simply the finite relative frequency measure.) Locality adds the requirement that these random variables be stochastically independent. The prism models avoid all this by restricting the domains of the response functions, so that only certain conditional distributions of them are well-defined. Conditional independence is preserved. The literature contains arguments, pro and con, as to whether stochastic independence is really necessary for locality to hold, in the intuitive sense of no action-at-a-distance. (See, for instance Clauser and Horne (1974) and Clauser and Shimony (1978), pro, and Suppes and Zanotti (1976), con.) In this section I want to suggest a specific sort of model, *the synchronization model*, where independence breaks down but where locality clearly seems to hold.[2]

The motivating idea is to consider what happens in a correlation experiment when particles in both wings have already passed their respective analyzers. In the case of ideally efficient detectors, one might suppose that both particles would be detected, giving rise to a coincidence count. But this supposition may easily fail. For the particles may be delayed differently in passing through their analyzers or in the subsequent journey to the detectors so as to be significantly retarded relative to one another, and thus they may fail to produce counts that are in coincidence. I shall refer to these various possibilities by saying that the particles are not *synchronized*. Whether or not particles are synchronized after passing their respective analyzers may well depend on the relative orientations of the analyzers. Thus it makes sense to introduce a *coefficient of synchronization* as follows:

$C_{ij}(\lambda)$ = the probability that a pair of particles of type λ will give rise to a coincidence count when the

A-analyzer is in its i^{th} position, and the B-analyzer is in its j^{th} position, given that both particles do pass their respective analyzers:

Suppose, now, that the response functions are defined for all λ. Then we can introduce probabilities as follows:

$P_A(\lambda, i) =$ the probability for a count in the A-wing, for an A-particle of type λ, when the A-analyzer is in its i^{th} position.

$P_B(\lambda, j) =$ the probability for a count in the B-wing, for a B-particle of type λ, when the B-analyzer is in its j^{th} position.

$P_{AB}(\lambda, i, j) =$ the probability for a coincidence count when a pair of type λ is emitted and the A and B-analyzers are, respectively, in positions i and j.

We have that

$$P_A(\lambda, i) = A_i(\lambda) \qquad \text{and} \qquad P_B(\lambda, j) = B_j(\lambda).$$

Stochastic independence requires that

$$P_{AB}(\lambda, i, j) = P_A(\lambda, i) \cdot P_B(\lambda, j) = A_i(\lambda) \cdot B_j(\lambda).$$

However, taking synchronization into account, it would seem more reasonable to suppose that

$$P_{AB}(\lambda, i, j) = A_i(\lambda) \cdot B_j(\lambda) \cdot C_{ij}(\lambda).$$

By a synchronization model, I mean one in which this last equation is satisfied for $C_{ij}(\lambda)$ not identically 1. Such a model must satisfy

$$P_{A_i} = \int A_i(\lambda)\rho(\lambda)\, d\lambda = 1/2,$$

$$P_{B_j} = \int B_j(\lambda)\rho(\lambda)\, d\lambda = 1/2 \text{ and}$$

$$P_{A_iB_j} = \int A_i(\lambda)B_j(\lambda)C_{ij}(\lambda)\rho(\lambda)\,d\lambda.$$

(The integration is over the whole λ-space L.)

It is not difficult to construct synchronization models. Using the devices of the preceding section, there is an especially simple way to do so. According to this way, there are two cases to consider; in both cases λ is supposed to be uniformly distributed on [0, 1].

Case 1: $P_{\theta'} < P_\theta$.

Suppose that

$$A_i(\lambda) = f(\lambda) \text{ for } i = 1, 2;$$

$$B_j(\lambda) = g_\theta(\lambda) \text{ for } j = 1, 2;$$

$$C_{11}(\lambda) = C_{21}(\lambda) = C_{22}(\lambda) = 1 \text{ for all } \lambda\text{; and}$$

$$C_{12}(\lambda) = g_{\theta'}(\lambda) \text{ for all } \lambda.$$

Then

$$\int_0^1 A_i(\lambda)B_j(\lambda)C_{ij}(\lambda)\,d\lambda = \int_0^1 f(\lambda)g_\theta(\lambda)\,d\lambda = P_\theta = P_{A_iB_j},$$

for $(i, j) = (1, 1)$, $(2, 1)$ and $(2, 2)$.

And

$$\int_0^1 A_1(\lambda)B_2(\lambda)C_{12}(\lambda)\,d\lambda$$

$$= \int_0^1 f(\lambda)g_\theta(\lambda)g_{\theta'}(\lambda)\,d\lambda = \int_0^1 f(\lambda)g_{\theta'}(\lambda)\,d\lambda = P_{A_1B_2}$$

Case 2: $P_\theta < P_{\theta'}$.

Suppose that

$$A_i(\lambda) = f(\lambda) \text{ for } i = 1, 2;$$

$$B_j(\lambda) = g_{\theta'}(\lambda) \text{ for } j = 1, 2;$$

$$C_{11}(\lambda) = C_{21}(\lambda) = C_{22}(\lambda) = g_\theta(\lambda) \text{ for all } \lambda\text{; and}$$

$$C_{12}(\lambda) = 1 \text{ for all } \lambda.$$

Then

$$\int_0^1 A_i(\lambda)B_j(\lambda)C_{ij}(\lambda)\,d\lambda = \int_0^1 f(\lambda)g_{\theta'}(\lambda)g_\theta(\lambda)\,d\lambda$$

$$= \int_0^1 f(\lambda)g_\theta(\lambda)\,d\lambda = P_\theta = P_{A_iB_j},$$

for $(i, j) = (1, 1)$, $(2, 1)$ and $(2, 2)$.

And

$$\int_0^1 A_1(\lambda)B_2(\lambda)C_{12}(\lambda)\,d\lambda = \int_0^1 f(\lambda)g_{\theta'}(\lambda)\,d\lambda = P_{\theta'} = P_{A_1B_2}.$$

Hence, in each case, the posited synchronization model gives the quantum mechanical probabilities.

5. CONCLUDING REMARKS

The prism models and the synchronization models show that the probabilities found in any correlation experiment designed to test the Bell inequalities can be accounted for in several different ways, without recourse to nonlocal mechanisms or processes. Of course, it may well turn out that the machinery of these models is itself subject to experimental test and that it will be disconfirmed. For example, very highly efficient experiments – of the sort suggested in Sec. 7.2 of Clauser and Shimony (1978) – may be able to rule out at least some of the possible prism models. Likewise, experimental data on transit times and retardations may show that the required coefficients of synchronization are not reasonable. It is precisely in order to generate this sort of criticism and discussion that I set out these ideas here. At this stage, however, one thing seems clear, and not acknowledged well in the recent literature. It is that the Bell-type arguments and the experiments which support them cannot be straightforwardly understood as arguments against, or experimental refutations of, locality. A great deal more than that is involved, and it is entirely possible that locality will survive this critical examination and that other principles – like the random variables framework – will go by the wayside instead.

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NOTES

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[1] Similar criticisms have been expressed by Lochak (1976) and by Bub and Shiva (1978). The best known hidden variables theory, that of de Broglie, does not employ this framework, and the group working on de Broglie's ideas seem quite clear in their rejection of it (1976). In work not yet published I have shown that the Bell (or, equivalently, the Clauser-Horne) inequalities hold if and only if the observables of the experiment can be represented as random variables (over a common space); i.e., if and only if there is a joint distribution function for all the observables (simultaneously). Thus the Bell results and the framework of random variables stand, or fall, together.

[2] The central point of my (1980) is to show, in considerable detail, why physical locality does not require stochastic independence at λ. The reader will also find there a different construction of deterministic synchronization models, and a discussion of one way to calculate the maximal prism model probabilities from experimental counting rates.

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