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Certain Matters in Relation to the Restricted Theory of Relativity, with Special Reference to the Clock Paradox and the Paradox of the Identical Twins.* I. Fundamentals

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The paper first discusses the significance of the meaning of space-time coordinates as attached to "events" in the Lorentzian transformation. Restricted relativity involves two distinct parts: A—the invariance of the *forms* of the laws under the transformation: B—the hypothesis that similar experiments performed in relatively moving frames S and S' give identical results. The test of A is a matter of pen and paper. B involves the hypothesis that the instruments are such that the actual measurements of space-time coordinates of events shall be related, for the two systems, by the transformation. Only by the postulation of a theory such as the quantum theory, but one relativistically invariant in sense A, can one understand the relationship between the instruments, whether the said instruments be constructed by independent observers in S and S' from the material around them, or whether the observer in S' acquires his instruments from S by setting them in motion. The second method of acquiring the instruments will not, in all cases, yield measurements related by the transformation. Thus, if we start with a system of *isolated* clocks which have been synchronized in S according to Einstein's principle, and if we transfer them to S', their "rates" (in a suitably defined manner) may be expected to alter in accordance with the transformation. However, it will remain for the observer in S' to synchronize the clocks in that new frame, having adjusted the time origin of one of them.

1. INTRODUCTION

MANY will feel that enough has already been written on the Paradox of the Identical Twins. However, the fact that distinguished authorities have differed so drastically in this matter[1] and that the reciprocal arguments seem, even yet, to be unsatisfactory to the contestants—these facts provide the reason why the present writer, at the risk of still further complicating the issue, ventures to add another paper to the list. He does so because, on considering the questions involved, it has appeared to him that several matters relating to the meaning of the restricted theory call for comment even at this time. As a result, it may be that what will here be written will transcend in interest the particular elements which have direct reference to the "Paradox." To deal exclusively with the minimum of matters necessary to the interpretation of the supposed "Paradox," even if successful in providing a conclusion which could stand—technically—firm against counter arguments, might still leave in the background a number of questions to provide a sense of unsatisfactoriness in the over-all picture. For this reason we shall, in Part I, avoid all reference to the "Paradox" itself, and concentrate our attention upon more fundamental questions, relegating the discussion of the "Paradox" to the position of a corollary to the more fundamental discussions to be discussed in Part II. Many matters to be dealt with will undoubtedly be well recognized by authorities on relativity theory, even though some of them may have been passed over lightly, if not completely, in current literature. For this reason, the present writer lays no general claim to originality in anything which may be stated, or insisted upon with emphasis. It is always difficult to conclude what attitudes may be in the *minds* of others, even though such attitudes may not have been expressed in writing. And so, with this apology, we shall proceed.

* Assisted by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.

[1] See, for example, H. Dingle, Nature **144**, 888 (1939); **146**, 391 (1940); W. H. McCrea, *ibid.* **167**, 680 (1951); C. Møller, *The Theory of Relativity* (Clarendon Press, Oxford, 1952), p. 48 and p. 258; Sir George Thomson, *The Foreseeable Future* (Cambridge University Press, New York, 1955), p. 89; H. Dingle, Nature **177**, 782 (1957); **177**, 785 (1957); W. H. McCrea, *ibid.* **177**, 784 (1957); J. H. Fremlin, *ibid.* **180**, 499 (1957); H. Dingle, *ibid.* **180**, 500 (1957); **180**, 1275 (1957); Sir Charles Darwin, *ibid.* **180**, 976 (1957).

SIGNIFICANCE OF THE RESTRICTED THEORY IN RELATION TO COORDINATES

2. The Importance of Events

The Lorentzian transformation of the restricted theory of relativity invokes a relationship between the coordinates x, y, z, t, of an event observed in a system S, and the coordinates x', y', z', t', of the same event observed in another system S', which moves relative to S with a velocity v parallel to the axis of x. The well-known relationship, with $\beta=(1-v^2/c^2)^{-\frac{1}{2}}$, is

$$x'=\beta(x-vt);\quad y'=y;\quad z'=z;\quad t'=\beta(t-vx/c^2). \tag{1}$$

The concept of an "*event*" is very important in this matter. If I strike a match, I create an event. If two bodies collide, that is an event. Observers in both S and S' will agree as to the actual *occurrence* of an event, although their measures as to where, and when, it occurred will be different. Strictly speaking, there is no meaning to the relationship between x, y, z, t, and x', y', z', t', except as applied to an event; and although it is not customary to emphasize the fact, and while there are cases where it appears to be ignored, it is always possible to interpret the use of coordinates as measures applied to an event. Thus, for example, when we express the velocity of a particle in S' in terms of the velocity in S, there appears to be no event involved. We can bring in the role of events in the following manner:

Confining ourselves, for illustration, to the axis of x, suppose that x, t corresponds to the birth of a fly, and $x+\delta x$, $t+\delta t$, to the death of a mosquito. Then, from (1),

$$\delta x'=\beta(\delta x-v\delta t);\quad \delta t'=\beta(\delta t-v\delta x/c^2)$$

so that

$$\frac{\delta x'}{\delta t'}=\frac{\delta x-v\delta t}{\delta t-v\delta x/c^2}=\frac{(\delta x/\delta t)-v}{1-(v/c^2)(\delta x/\delta t)}. \tag{2}$$

The result (2), while true, has no interesting significance. $\delta x/\delta t$, and $\delta x'/\delta t'$ are not related to anything which is of interest.

Suppose, however, that while the event at x, t corresponds to the birth of the fly, $(x+\delta x)$, $(t+\delta t)$ corresponds to the death of the mosquito *resulting from* its being killed by the fly. Then $\delta x/\delta t$ and $\delta x'/\delta t'$ are very interesting quantities. They correspond to the velocity of the fly as seen in S and S', respectively, and (2) assumes a very fundamental interest as the equation of transformation for a velocity. It is not to be forgotten, however, that it acquires this meaning only on account of the possibility of relating the situation as a whole to events.

3. The Significance of Coordinates of Space Time in General

The concept of scales and clocks as means of obtaining the magnitudes of the coordinates of events is, in the last analysis, very primitive and naive unless we recognize at the outset, that these concepts are usually purely symbolic. Nobody sticks scales and clocks throughout space to determine the space-time coordinates of astronomical events. One makes measurements with instruments attached to the earth, and by processes of calculation, involving a certain amount of subsidiary theory, the desired magnitudes of the coordinates are secured. The hypothetical readings of scales and clocks are no more than the symbols of this whole procedure—part measurement, part theory, part computational—by which the magnitudes of the space-time coordinates of event[2] are to be ascertained.

4.

In the last analysis, the story of physics is largely the story of "events." It is *qualitative* in nature, and the end point of theories—such as the theory of gravitation—is to assert that if certain events occur, then others will be found to occur. In seeking to formulate a theory in this sense, it is convenient to introduce space-time coordinates as a scaffolding for the purpose of symbolizing an event—the collision of two astronomical bodies, or the blackening of a certain designated grain on a photographic plate by a spot of light (the image of the planet). The event is symbolized by equality of the space-time coordinates of the two participants in the event—for example, the photographic grain and the spot of light. In the equations associated

[2] In case one questions the meaning of the "event," in the case of the observation of the position of a planet and the corresponding time, the answer is that the event is the coincidence of the image of the planet with the cross hairs of the observing telescope.

with such a theory, the space-time coordinates occur as mere symbols, the equating of corresponding symbols for two entities symbolizing the corresponding event. The coordinates do, therefore, play a part in expressing the *forms* of the laws. However, as regards observation, it is possible to deal exclusively with "events," and indeed, it is possible to show how such a law as the Newtonian Law of Gravitation could be discovered from the mere observation of the *occurrence* of events, without the necessity of making any *measurements* of magnitudes of coordinates, rates of change of coordinates, etc., at all. However, this matter will not be pursued further here, since it has no immediate bearing upon the main purpose of this paper.[3]

5. The Significance of Space-Time Coordinates in Relation to the Restricted Theory of Relativity

It is not always realized that what is comprised under the restricted theory of relativity involves two distinct postulates A and B, as follows:

A. The Laws of Nature as Represented by the Appropriate Differential Equations are Invariant in Form under the Lorentzian Transformation

The test A is a matter of pen and paper, used in the light of the appropriate definitions of the symbols involved.[4] It does *not necessitate* the conclusion that similar experiments performed on systems moving with relative rectilinear constant velocity shall give identical results. In the last analysis, the *performance of an experiment* means the assignment of those magnitudes which it is necessary to assign to cause the differential equations to give a unique solution. Again, by performing the *same experiment* in two systems, we mean the assignment of the same magnitudes in the corresponding places in each of the systems concerned. Roughly speaking, we must assign the same magnitudes for the corresponding initial conditions in the two systems.

B. Similar experiments Performed in Systems S and S' Moving Relatively with Constant Velocity Give Identical Results

As already stated, the test of A is a matter of pen and paper, but B implies something else. Accepting A, the necessary and sufficient condition for B to hold is that the measuring instruments in S' shall be such that the values which they give for the space-time coordinates of an event shall be related by the Lorentzian transformation to the values given for the same event by the instruments in S.

6.

The customary approach to this matter is one which pictures a state of affairs in which a set of instruments (scales and clocks) in S is given a velocity as measured in S, by which the instruments are transferred to S'. It is then inferred that as seen in S, certain changes are produced in the instruments so that measurements by them for an event are related to those of a duplicate set remaining in S by the Lorentzian transformation. It is indeed possible to discuss matters in this way; but the desired inference can only be substantiated by the invocation of another hypothesis, and if this hypothesis is invoked, the whole relationship between the instruments in S and S' assumes a much cleaner aspect, and one in harmony with the situation as it presents itself in nature, where I do not expect an observer on the moon, which moves relatively to me, to steal my apparatus from my laboratory on earth, set it in motion to accompany him on the moon, in order that he shall be able to carry out his scientific investigations to the end of discovering the laws of nature. I expect the observer on the moon (the system S') to construct his measuring instruments from the things which are around him, and it is my hope and expectation that he will construct instruments[5] whose measures of coordinates of events are related to my measures of the coordinates of the events in the manner defined by the Lorentzian transformation. And what is the basis of my hope and expectation that

[3] A paper on the matter is in process of preparation.

[4] Thus, sometimes the laws involve symbols other than the coordinates and their derivatives, symbols such as electric and magnetic fields. Here, the meaning to be attached to the transformation of the fields is only to be found in the light of their definitions, which ultimately stem back to matters concerned with the positions and motions of points specified by the coordinates.

[5] Here again, we regard the use of instruments—clocks and scales—as only symbolic of a well-defined procedure of attaching space-time coordinates to the events as discussed in Sec. 3.

this shall be so? The basis is to be found in the hypothesis that there *exists* in nature some additional set of principles of correlation of phenomena, principles such as are involved in the quantum theory, principles invariant under the Lorentzian transformation in the sense of hypothesis A. The test of this hypothesis A is a matter of pen and paper.

Suppose that, in the light of these considerations, I have certain apparatus which I intend to use for the measurement of space-time coordinates of events. Suppose that I know enough about this apparatus to be able to write down what the quantum theory student would call the Hamiltonian for it. Then, in principle, I can set up, for example, a time-dependent Schroedinger equation for it. The x, y, z, t, in this equation must, for the time being, be regarded merely as letters, and not necessarily as magnitudes measured with scales and clocks. Out of the development of the consequences of the quantum theory I shall find that certain events happen[6] as a function of t, with certain probabilities which, in macroscopic cases amount, usually, to certainties. A sequence of such events might, for example, be secured by a "radium clock," in which a source of radioactivity is allowed to charge an electroscope until the leaf is deflected to a position where it grounds itself, after which the charging process repeats itself *ad infinitum*.

Now up to this point, I really have no physical meaning for t. It is simply a letter in my equations. However, my mathematical relationships will provide me with a formal symbolic relationship between N, the number of events, and t. If now *I decide to measure time* by the number of a certain class of such events which have occurred, I establish a numerical meaning for t through the mathematical relationships aforesaid between t and the number of events of the specified class. Suppose I choose a sequence of events and define t as proportional to N. Then, if these events are properly chosen, they will provide a measure of the kind of time which we want for our purposes.[7] By analogous, but rather simpler procedures, we can provide a meaning for x, y, z, in terms of our apparatus. Perhaps the simplest procedure for defining a length is to imagine a light signal sent from one end P of a rod and reflected by a mirror from the other end, Q. If $_{P\ P}$ is the interval between departure and return, measured by a clock designed as above, the distance l is defined as

$$l \equiv ct_{PQP}/2. \tag{3}$$

In accordance with Einstein's definition of simultaneity, clocks spaced along the axis of x are synchronous in their readings if, for a light signal departing from $x=0$ and $t=0$, the clock at x records the time $t=x/c$ for the arrival of the signal, where x is supposed measured in accordance with the definition implied in (3). We shall call the piece of apparatus established as above for use in the system S, the apparatus A_s.

Having now established our apparatus and its use for measuring space-time coordinates of events in the system which we shall continue to call S, and realizing that the apparatus is governed in its behavior by the quantum theory as stated above, the supposed mathematical invariance of the quantum theory under the Lorentzian transformation tells us that *another piece of apparatus can exist in S* (*where indeed it is in motion*), having events whose values for x', y', z', t', (as given by the transformation) have the same magnitudes as do the values of x, y, z, t, for corresponding events in the apparatus A_s. We shall call this new apparatus $A_{s'}$. Thus representatives of the radium clocks in A_s will be found in $A_{s'}$, as will representatives of the scales. If, using the apparatus A_s, an observer measures the space-time coordinates of an event, and using

[6] Such events could be atomic transitions associated with half-lives, etc. In the last analysis, even the events associated with the movement of the hand of a clock are interpretable as a series of events, related in a complicated way to more fundamental events such as atomic transitions. Incidentally, we have a "time-dependent" situation, a situation moreover, bound up with the fact that when the clock has "run down" it will cease to operate.

[7] This rather involved statement is really concerned only with the fact that if, for example, I measured time by a clock whose temperature is varying in a random manner, the kind of time I measure will be unsuitable for participation in simple form for any laws having an expectation of success in representing the laws of nature. We require no deep study of relativity or quantum theory to tell us that it is part of our duty as experimentalists to measure time in a manner which will lend itself most suitably to our purposes. Thus, laws which were mathematically invariant under the Lorentzian transformation with time measured in one way and denoted by t, would not, in general, be mathematically invariant if the time τ were measured in some other way, such that the new time was related to the old by $\tau = at^2$, for example.

$A_{s'}$, another observer measures the space-time coordinates of the *same* event, the two observers will find that their measures are related by the Lorentzian transformation. Strictly speaking, as far as hypothesis A alone is concerned, we have so far assigned no meaning to v in this transformation other than that it is a quantity less than c. However, looking at the equations of transformation, we see that $x'=0$ when $x=vt$, so that v represents the velocity of the origin in the apparatus $A_{s'}$ as measured by the apparatus A_s. In other words, the apparatus $A_{s'}$ is that of an observer in what we have called the frame S' which moves relatively to S with velocity v.

Now, of course, we have no guarantee that the physicist in S' will choose his apparatus according to the above principles. He may measure times by clocks which run in completely crazy fashions in relation to the clocks of the apparatus A_s. He may, with such apparatus, discover laws of nature, but they will not be of the same form as those obtained by use of the apparatus $A_{s'}$, although they will be transformable to that form by a transformation which eliminates the supposed craziness of the apparatus. All we can assert is that it is *possible* for the observer in S' to make apparatus which will yield space-time coordinates in the fashion envisaged for $A_{s'}$; and if the observer uses the same *kind* of principles for constructing his apparatus that the observer in S uses for constructing the apparatus A_s, he will certainly[8] arrive at the apparatus $A_{s'}$.

7. Effect on the Apparatus of Actual Impartation of Motion

In the section immediately preceding, we have seen how the observer in S' will be led to discover apparatus which leads to space-time measurements in harmony with the Lorentzian transformation, and without the necessity of borrowing apparatus from S. We now ask, as a matter of philosophic interest, whether, if we have in S two identical pieces of apparatus A_s, and set one in motion with velocity v relative to S, it will experience, as observed in S, such alterations as will cause it to *become* the apparatus which we have designated as $A_{s'}$. The apparatus will *certainly* alter as viewed in S. For when it was in S as A_s, it was a structure conforming in its behavior to the principles of the quantum theory; and since, by hypothesis, the fundamental equations of the quantum theory, even though we may not yet have discovered them, are invariant under the Lorentzian transformation, and are not, therefore, invariant under a simple Galilean transformation, it is manifest that the apparatus will not act in accordance with the principles of the quantum theory when set in motion in S without experiencing alterations. It certainly *will* conform to quantum theory requirements if it changes to the piece of apparatus we have called $A_{s'}$. But will it do this? In general, it will not. If we set it in motion with a sledge hammer, we shall probably dent one end of the apparatus permanently, and it certainly will not conform to $A_{s'}$. If, however, we set it in motion by forces which conform to what, in quantum theory principles, are known as small perturbations, then there is sense to believing that after the uniform motion is established, the apparatus will conform to the type $A_{s'}$. A complete discussion of this matter becomes very involved and will not be attempted here.[9] It will suffice to say that the essence of the matter is to the effect that the forces which set the apparatus in motion must not be such as to result in finite probability of *finite* transitions in energy states, etc. In other words, we must not hit the apparatus such a blow as will produce changes analogous to the breaking of chemical bonds. If the motion is not imparted in such a manner, we may say that the apparatus is set in motion "gently," this, indeed, being the ultimate meaning of the word gently. However, even with this provision, some reservations are necessary in regard to what is involved in

[8] It is, of course, possible that the observer in S' constructs apparatus which measures lengths in yards instead of in centimeters, and time in years, instead of seconds. Such differences are obviously trivial, and involve only a change in units.

[9] These matters have been discussed by the writer in a paper: "Relativity, the Fitzgerald-Lorentz Contraction, and Quantum Theory" [Revs. Modern Phys. **13**, 197 (1941)]. In a much earlier paper, the writer has discussed the question of the Lorentz Contraction and associated matters in terms of the ideas of that pre-quantum theory epoch. See "The Fitzgerald-Lorentz Contraction, and an Examination of the Method of Determining the Motions of Electrons when Considered Simply as Singularities, Moving so as to Satisfy the Electromagnetic Scheme" (Phil. Mag. **23**, 86–95 (1912)].

setting the apparatus in motion. These will be discussed in the following section.

An important conclusion to be drawn from the previous discussion is to the effect that, in general, and with certain reservations to be discussed in Sec. 8, matter, when set into uniform motion as measured in a system S, does in actuality experience changes as seen in S, changes in harmony with the mathematical transformation; and these changes are just as "real" as any other changes which might be produced and observed in S, by any other operations such, for example, as by heating the matter.

In spite of the foregoing considerations, it is to be realized that the natural way for the observer in S' to acquire his apparatus is to construct it himself on the bases outlined in Sec. 6, and not acquire it by the more vulnerable process of setting in motion apparatus belonging to S.

8. Certain Peculiarities Relating to the Effects of Imparting Uniform Velocity to Matter

Let us seek to form a picture of what happens when a combined space-time measuring device is transferred from S to S' by the impartation of a velocity v. We shall consider two extremes of adaptability of such a device to the representation of the space-time system of measurement in S.

The first device is represented by a rod whose atoms are arranged equally spaced along the axis of x, each being composed of a central nucleus with an electron in an orbit which, in S, is circular, or approximately so. Figure 1 represents two such devices, both as yet in the frame S, so that they are both alike and so that their electrons lie in exactly similar positions in their orbits. These electrons, by the angles which their radius vectors make with the axis, symbolize equal times for all values of x. The electrons are our clocks.[10]

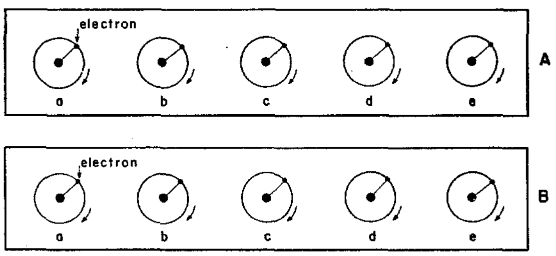

FIG. 1.

Recalling the transformation (1), *viz.*,

$$x' = \beta(x - vt); \quad t' = \beta[t - (v/c^2)x] \tag{4}$$

and the reciprocal transformation

$$x = \beta(x' + vt'); \quad t = \beta[t' + (v/c^2)x'] \tag{5}$$

we see that, following the impartation of the velocity v to the system as observed in S, the situation is as represented in Fig. 2 when observed in S, where B remains, as before, in S, while A (now A') has experienced a real change as observed in S.

Referring to (4) we see that, if two events are simultaneous in S and occur at a distance apart equal to l, this distance will be equal to $\beta^{-1}l'$, where l', is the measurement in S'. Truly, the events will not be simultaneous in S'. However, let us introduce the concept of two continuously occurring series of events, one set occurring at one end of l' and the other occurring at the other end. The events could be the successive crossing of fixed points in S' by electrons in the first and last orbits of A', Fig. 2, the crossings corresponding to the maximum value of the electron's x' coordinate in its orbit in each case. The constant distance l', while not representing the distance between two events occurring *simultaneously* in S', does represent a well-defined constant quantity representing the distance between *some* pairs of events, and a quantity related to l by $l = \beta^{-1}l'$. Thus our rod, which was A in S, will now appear as A' when set in motion if it is observed from S. It will be noted that the orbits, which were circular in S, are now

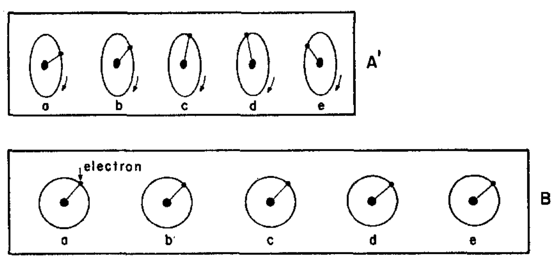

FIG. 2.

[10] This state may be said to represent synchronization of the clocks according to some principle of synchronization, such, for example, as the principle adopted by Einstein.

ellipses in S. However, the electrons in these ellipses will not occupy, at constant time in S, similar positions in their orbits as they do in B of Fig. 2. Suppose that the electrons' orbits represent dials with numbers attached, the number reading corresponding to the position in which the electrons find themselves, then the picture, Fig. 1(A), will go over into Fig. 2(A′). We see from (4) that, for a constant dial reading, t_0, for the dials in B, Fig. 2, the dial reading t' in A' will be given by $t'=\beta t_0-(\beta v/c^2)x$. The quantity x is composed of a large part representing the distance of the electron from the origin, and a small part representing the distance from the center of its orbit. All of this is, of course, known to the most elementary student of relativity. It has been necessary to recall it, however, in order to provide a basis for the next comment, which is as follows: We note that in being transferred from A to A', the clock c, for example, has experienced two things—a change in rate, as exemplified by the fact that, as seen from (5), $dt'/dt=\beta^{-1}$ at constant x', and a further change in the absolute value of time depending upon x. Even at the instant of transfer—say $t=0$, the clock c, for example, experiences a sudden change in absolute reading, and the other clocks all experience sudden changes of this kind, and *to extents depending upon x*. Only the clock a appears to have experienced no change of this kind, and this only results from the way in which we have drawn the figure. Indeed, even if we should be content to accept the alteration in clock "rates," this *sudden* change is something rather surprising. The matter in question must not, of course, be confused with any reference to absolute time; for since the transformation was *determined* by the condition that observers in S and S' both measure the same value for the velocity of light by examining the time interval between two events separated by a certain distance, the events being initiated by the passage of a light wave over—let us say—two matches fired by the passage of the wave, it is obvious that such a determination can have no relation to any *absolute* origin of time in either S or S', or to any relation between the space origins in these two systems.

We thus have the curious situation, that at the instant of transfer, the various clocks move their positions, possibly by many thousands of miles, change their rates, and experience a sudden alteration in dial reading recorded. Such a situation is not as remarkable as it may seem to be, since all parts of A_s constitute a single system. It becomes less remarkable when viewed from the standpoint of the discussion in Sec. 7, in which it is recognized that, in general, restrictions are imposed on the relationship of the parts of A_s— restrictions demanded by the quantum theory—and in which it is further recognized that in changing from A_s to $A_{s'}$ the apparatus has continually to adjust itself to the quantum theory requirements. However, a real logical problem faces us when, for example, A_s comprises a series of clocks *completely separated* in A or B, in S, i.e., separated in the sense that there are no forces between them. What am I to mean by setting this set of clocks in motion? Am I to apply forces to one clock, leaving the others to take care of themselves? Truly, they will find it a difficult task to conform to a condition represented by the Lorentzian transformation. However, to minimize the difficulty of attaching meaning to the problem, suppose I act on each individually, so that I do in actuality realize a condition in which they are all going along with a velocity v as observed in S. Is there anything to force them to come closer together as viewed from S; and is there anything to provide for a condition in which, not only are their rates changed, but also their actual readings at $t=0$, the time when the velocity was imparted, are changed, and to an extent depending upon x? Here we see the real weakness in relying on the act of "setting the apparatus in motion" to provide us with the measuring instruments specified by $A_{s'}$. It is true that in certain types of apparatus of such a nature that the quantum theory takes a firm hold on the behavior of the different parts, the act of "setting the apparatus in motion" will indeed provide the system $A_{s'}$, complete in all its aspects. When the apparatus is not governed in this over-all manner by the quantum theory, something is left for the observer in S' to do.

9.

Now in the light of the above, is there anything which we can say about the effect of motion on the instruments, and which does not call for further adjustment on the part of the observer in S'? The answer is that there is indeed one thing, which may be summed up in the following statement:

Suppose that, in S, we have a set of clocks synchronized in the manner already defined in Sec. 6, and suppose that we have, at some value of x (say $x=x_0$) an additional clock b, which is just like its companion of the aforesaid set which stands at $x=x_0$. Suppose we set b in motion with velocity v as measured in S, so that it is now in the frame S', occupying a position which is constant in S'. Then b will alter in such a manner that, if $(T_1')_b$ and $(T_2')_b$ are the times which it records for two events occurring *at constant* x', i.e., in two different positions which it occupies in S and if T_1 and T_2 are the corresponding times recorded by the appropriate two clocks of the set belonging to S, then the intervals $(T_1')_b - (T_2')_b$ and T_2-T_1 are related by

$$(T_2')_b-(T_1')_b=\beta^{-1}(T_2-T_1). \qquad (6)$$

This is something which our transformation suggests, which our hypothesis B demands, but for which neither gives a reason. Any reason is to be sought in the existence of an invariant quantum theory or its equivalent. It will be noticed that *nothing* is said about any relationship between the actual clock-value of the time for an event as seen in S and the actual clock-value of the time for that event as seen in S'. Only the *intervals* are related by (6).

In the light of the above, what remains for the observer in S' to do in order that he shall provide himself with a set of instruments which will measure for the space-time coordinates of events the dashed values *defined* by (1)? *The answer is that the said observer must synchronize his clocks*, and this, apart from a triviality, is all that he need do. The reason will be understood from the following:

Suppose, in the system S, we start with a set of points distributed throughout the space. At each point there shall be a clock, all clocks being alike in structure. We send a light signal from P to a mirror at Q, which mirror reflects it back to P. The interval between departure and return shall be t_{PQP}. *We then define the length l between P and Q* in accordance with (3) by

$$t_{PQP}=2l/c$$

where c is a constant to be assigned. Having assigned c we can determine l. Or, if we wish to assign l in terms of marks on a specified rod, we can determine c. If we wish to graduate our clock dial so that it reads in what we call seconds, and if our scale is marked out in what we call centimeters, c will naturally turn out to be 3×10^{10}.

In this manner, we map out all distances, and in particular, distances parallel to the axes. It will be noted that in these operations, only *one clock* is used for the measuring of a single distance, and *so far all the clocks are unrelated in actual readings.*

Recalling that, for any pair of points, P and Q, t_{PQP} involves measurements on only one clock, suppose that we adjust the actual readings of the clocks so that, if t_P and t_Q are the times of departure of the light from P and arrival at Q, as recorded by the clocks, the relation

$$t_{PQP}/2=t_Q-t_P \qquad (7)$$

holds. *Then in accordance with Einstein's definition of simultaneity, we say that the clocks are synchronized in S.* Suppose two events, one at P and the other at Q, are initiated by a light beam which travels from P to Q. Suppose that t_Q-t_P is the time difference of the events as recorded by the aforesaid synchronized clocks in S and l_{PQ} is the space difference measured in the manner described above. Then

$$l_{PQ}=t_{PQP}/2c=(t_Q-t_P)/c$$

where again we emphasize that t_{PQP} is measured by a single clock, while t_P and t_Q are measured by different clocks. Recognizing that

$$l^2{}_{PQ}=(x_Q-x_P)^2+(y_Q-y_P)^2+(z_Q-z_P)^2$$

where x_Q, x_P, etc., are coordinate distances from an orgin, we have

$$(x_Q-x_P)^2+(y_Q-y_P)^2+(z_Q-z_P)^2 = (t_Q-t_P)^2/c^2. \qquad (8)$$

We now inquire what numbers x', y', z', t', must be assigned in S' to an event as corresponding to the assignment x, y, z, t, in S for the same event, if the relation (8) is to be invariant. The result is, of course, represented by the well-known relations (1). *So far x', y', z', t', have no meaning other than that given by the transformation (1).* However, let us see what properties are possessed by times and lengths as given by the transformation for S', and see to *what extent they correspond to the times and lengths which the observer in S' measures with the apparatus which he has obtained from S by the process of setting that apparatus in motion with velocity v.* The latter we shall designate by capitals, for example, T' for time.

In the first place, the dashed lengths and times given by the transformation conspire to yield for light the velocity c regardless of the direction of the light. Thus l', *as given by the transformation* for the distance between P and Q, satisfies the relation

$$l' = (t_Q' - t_P')c \tag{9}$$

where t_P' and t_Q' are the times of departure of a light signal from P and its arrival at a mirror at Q, *these times being given by the transformation.* By the same token, for the return journey

$$l' = (\bar{t}_P' - t_Q')c \tag{10}$$

where $\bar{t}_P'$ is the time of return of the signal as given by the transformation. From (9) and (10),

$$(\bar{t}_P' + t_P')/2 = t_Q' \tag{11}$$

which shows that the "clocks" yielded by the transformation are synchronous in S'. As a consequence any actual clocks which are to give the times given by the transformation *must be synchronous throughout the space of S'.* Thus, the readings $\bar{T}_P'$, T_P', T_Q' of such clocks must satisfy the condition

$$(\bar{T}_P' + T_P')/2 = T_Q'.$$

Again writing

$$t_{PQP}' \equiv \bar{t}_P' - t_P'$$

where t_{PQP}' is the interval of time between departure from P and return to P as corresponding to the transformation, we have

$$(2l'/c) = t_{PQP}'. \tag{12}$$

Now, in analogy with (3), the length L' between P and Q is *defined by the observer in S'* as a quantity given by

$$2L'/c \equiv T_{PQP}' \tag{13}$$

where T_{PQP}' is the time interval as *measured by the* actual clock at P which was obtained by setting in motion one of the duplicates belonging originally to S. Since the clock is at a constant position in S', our hypothesis as to the alteration which this clock has experienced as a result of having the motion imparted to it results, from (6), in

$$T_{PQP}' = \beta^{-1} t_{PQP}$$

where t_{PQP} is the interval observed in S—not now on the same clock—between the two events corresponding to the departure of the light from and its return to P, the clocks in S being synchronized. However, it results from the transformation that

$$t_{PQP}' = \beta^{-1} t_{PQP}.$$

Hence $t_{PQP}' = T_{PQP}'$, so that the definition of L' given by (13) results in the same magnitude as the value l' given by (12) and corresponding to the transformation. Moreover, since the actual clocks in S' are synchronized by the observer, as aforesaid,

$$T_{PQP}' = 2(T_P' - T_Q'),$$

and since $t_{PQP}' = 2(t_P' - t_Q')$ and $T_{PQP}' = t_{PQP}'$, we have, for all pairs of points,

$$T_P' - t_P' = T_Q' - t_Q' = \text{const.}$$

Thus, the times given by the actual synchronized clocks in S' are the same as those given by the transformation, except for an irrelevant constant.

Thus, in concluding the discussion of this matter, we realize that if the clocks in S' are to be those "borrowed" from S, and set in motion, then in order that these clocks shall give, for events throughout the space of S', times which are the same as those given by the transformation,

1. The motion imparted must alter their *rates* in the sense defined by (6).
2. The observer in S' must adjust[11] the dial in S' which stands at $x=0$ to read $t'=0$ when $t=0$, and he must then synchronize his dials in the sense defined by

$$\tfrac{1}{2}(\bar{T}_P' + T_P') = T_Q' \tag{14}$$

[11] This adjustment is a trivial matter. The important thing is the synchronization.

as the hypothetical clocks yielded by the transformation are synchronized by (11). This law, of course, is Einstein's law of synchronization.

As regards lengths, all that is then necessary is that they shall be defined by similar equations (3) in S and (13) in S'. If one is to construct a measuring rod, he must construct it on the basis of this definition.

Finally, we may assert, as already stated, that if we invoke the quantum theory in conjunction with the assumption, for that theory, of invariance in mathematical form under the Lorentzian transformation, we can provide reasons why clocks and scales will look after themselves in all respects when set in motion, and will give measures in harmony with the transformation. While the invocation of this idea for a connected system like a *rod* is reasonably acceptable, it has its limitations in the case of clocks which are "isolated" from one another and yet are to constitute the clocks in S', the limitations being concerned primarily with the synchronizing action.

Flash Period of 1958 Delta I

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Careful observation of the flash period of the artificial satellite, 1958 Delta I (the rocket case of Soviet Sputnik III), was made at Ames, Iowa, from July 21 through November 29. The flash period increased steadily during this time until approximately one-half week before fall-in. This indicated a slowdown in the rate of tumbling of the rocket case. During the last few days of life the flash period decreased, indicating a speedup in tumbling rate. What causes this change? Future satellite observers are urged to observe this phenomenon to see if it is general.

WHEN 1958 Delta I was first observed by the group at Iowa State, its flashing was recognized as the expression of a tumbling of the satellite. For this reason it was decided to keep an accurate record of the flash period. The expectation was that the period would slowly increase, and it was hoped that good data might help (we did not know just how) to give information on the density of the atmosphere at these very high altitudes.

Observations were made consistently on almost all visible passes from July 21, 1958, to November 29, 1958, from latitude 42.02865° N longitude 93.65027° W (Electrical Engineering Building, ISU, Ames, Iowa).

A magnetic tape recorder was used to make the record. One member, Mrs. Carr, of the Iowa State University observing team was assigned the duty of making the flash record. When the satellite was judged brightest in each cycle, that person yelled "bright" or tapped a tap-bell. This and all other operations were recorded on the tape along with time signals by radio from WWV or CHU. When the tapes were read, each event was recorded in the notes to the nearest second. Only occasionally were time interpolations attempted. Psychological lag or reaction time was not corrected for. Since this lag is always in the same direction, it should affect absolute times but have only minor effects on period determinations.

July 21 through 23 gave flashes which were alternately bright and not so bright. The photographs showed the effect and indicated that it was evident when the satellite was near the point of closest approach. Later, this effect was not obvious, if present. This phenomenon indicates that the flash period is one-half the tumble period.

The record is given in Table I and Fig. 1.

Column 1, headed Time, gives date and time of observation in Universal time.

Column 2 gives n, the number of flash periods observed (number of flashes less one). Occasionally a flash was not observed because of local cloud or was not recorded because of confusion